A Loewner-based system identification and structural health monitoring approach for mechanical systems

Please cite [1-2], alonside the original development [3-5] of the Loewner Framework, when using this software for your work or research, Thank you.

This tutorial is the MATLAB code for "A Loewner-based system identification and structural health monitoring approach for mechanical systems" article [1].

The model

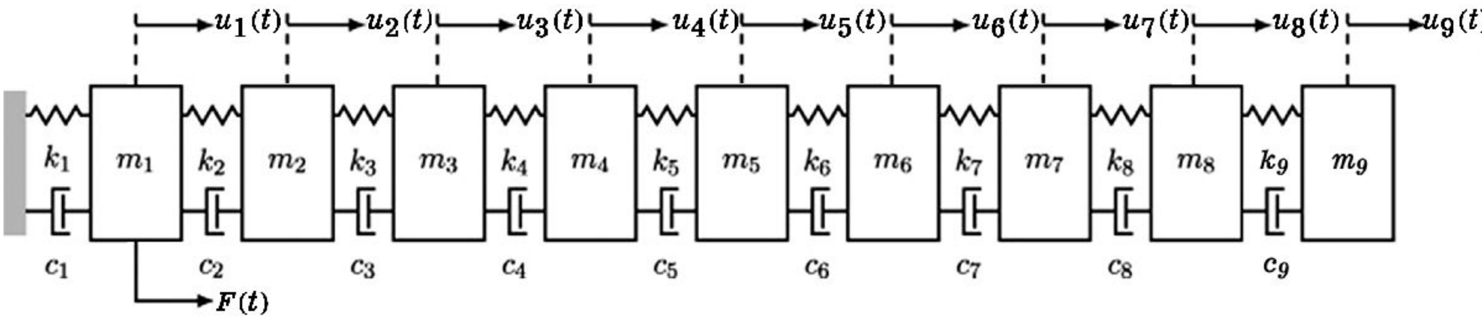


Figure 1. 9 DoFs system: schematic diagram (Retrieved from [1]).

The tutorial will consider the 9 DoFs system used in the main paper to illustrate in this tutorial how the MATLAB's implementation of the Loewner Framework (LF) works. The system, is made of nine masses, such that $m_n = 1kg$ each, adjacently linked with springs,such that $k_n = 5000Nm$, and dampers, characterised by the critical damping ratio of $\zeta_n = 1\%$ at all nodes. The numerical model was excited with a Unit ($1N$) rectangular impulse applied to the first mass as input force, the data was recorded at a sampling frequency $f_s = 200Hz$, with a frequency resolution of $\Delta_f = 0.05Hz$. Let's import the FRF and the numerical modal parameters of the above mentioned system:

```
clear all
close all
load data_9dof %load dataset n by m by q
```

n is the number of frequency points, $100/.05$ in our case

m is the number of channels in the FRF, 9 in our case

q is the dataset considered, 4 different datasets are attached to this tutorial. Namely, the undamaged case and three damaged case, where the stiffness of the fifth element has been reduced, respectively, by 50%, 30%, and $\%10$.

System Identification

Let's now fit the LF to the FRF of the undamaged system

```
%[id,model,fit] = LF_id(FRF,si,nn)
[id,model,~] = LF_id(FRF(:,:,1),f,18); % This is the LF function
```

```
Starting parallel pool (parpool) using the 'Processes' profile ...
Connected to the parallel pool (number of workers: 6).
```

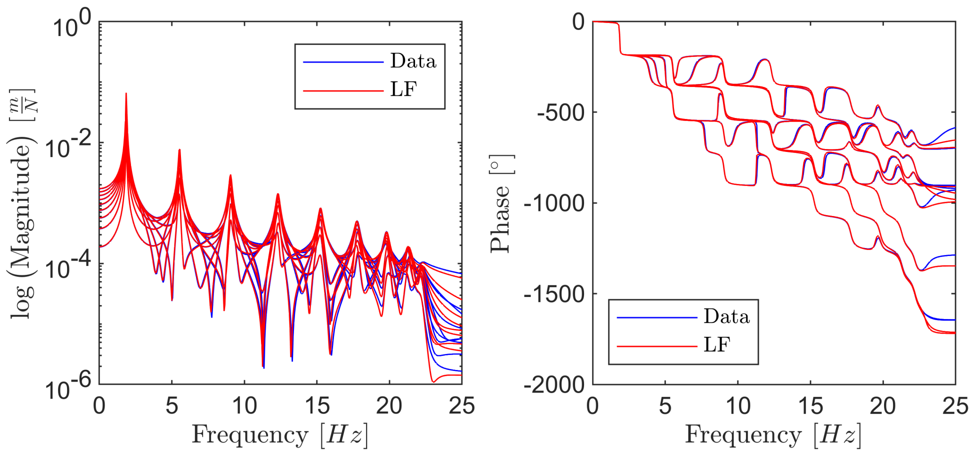
```
for jj = 1:length(f)
        FRFfit(:,jj) = model.C*( ((f(jj).*1j).*model.E-model.A)\model.B );
end
```

As explained within the function itself, FRF is where the FRF of a SISO or SIMO in the form of m by n or n by m n needs to be plugged in, si is the input frequency vector and nn the order. Since we know there are 9 modes within the examined interval, the order was set to 18.

Let's plot the amplitude and phase of the FRF and the model obtained via LF:

```
figure(1)
subplot(1,2,1)
p1=semilogy(f,abs(FRF(:,:,1)),'b','DisplayName','Data');
hold on
p2=semilogy(f,abs(FRFfit),'r-','DisplayName','LF');
xlim([0 25])
ylim([10^(-6) 1])
xlabel('Frequency [$Hz$]','Interpreter','latex')
ylabel('$\log \big($Magnitude$\big)$ [$\frac{m}{N}$]','Interpreter','latex')
legend([p1(1) p2(1)],'Location','northeast','Interpreter','latex')
axis square

subplot(1,2,2)
p3=plot(f(:),unwrap(angle(FRF(:,:,1)),[],1).*180/pi,'b','DisplayName','Data');
hold on
p4=plot(f,unwrap(angle(FRFfit(:,:)),[],2).*180/pi,'r-','DisplayName','LF');
xlim([0 25])
legend([p3(1) p4(1)],'Location','southwest','Interpreter','latex')
xlabel('Frequency [$Hz$]','Interpreter','latex')
ylabel('Phase [$^{\circ}$]','Interpreter','latex')
axis square
```

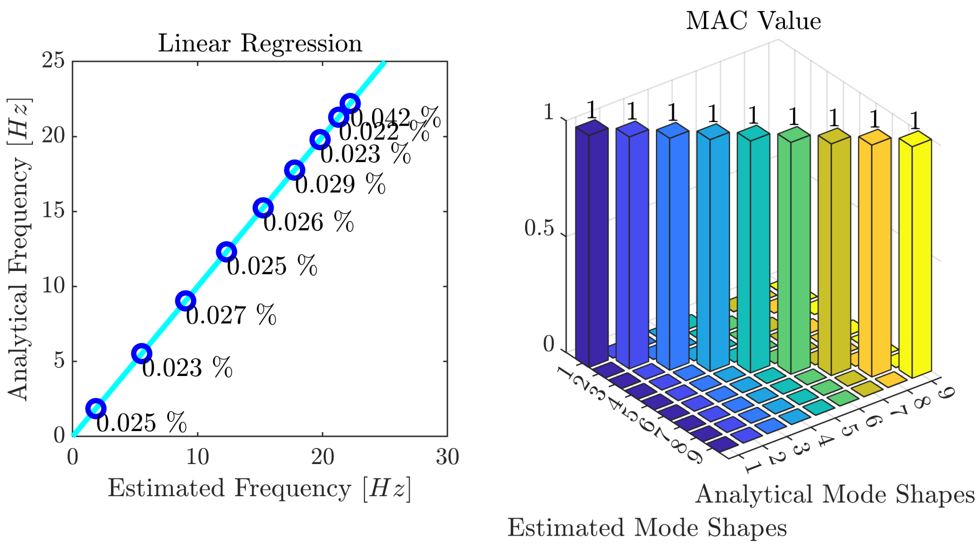


The plot shows a good correlation between the real FRF and the LF fit, both in term of amplitude and phase. Now, let's investigate the goodness of the modal parameters identification against the values computed from the numerical model. A linear regression will be used to represent the goodness of the evaluation of the natural frequencies and the MAC value for mode shapes coherence..

```
% plot the comparison for natural frequencies and mode shapes
%natural frequencies linear regression
figure(2)
subplot(1,2,1)
labels='$'+string(round(100.*(id.ident(1,:)-reference(1,:,1))./reference(1,:,1),3))+'$ $\%$';
plot([0,25],[0,25],'c','LineWidth',2)
hold on
plot(id.ident(1,:),reference(1,:,1),'ob','LineWidth',2)
text(id.ident(1,:),reference(1,:,1),labels,'VerticalAlignment','top','HorizontalAlignment','le
hold off
axis square
title('Linear Regression','Interpreter','latex')
xlabel('Estimated Frequency [$Hz$]','Interpreter','latex')
ylabel('Analytical Frequency [$Hz$]','Interpreter','latex')
set(gca,'TickLabelInterpreter','latex')

% mode shapes MAC value
subplot(1,2,2)
mac = compute_mac(reference(3:end,:,1),id.ident(3:end,:));
labels=string(round(diag(mac),2));
bar3(mac)
```

```
[X,Y] = meshgrid(1:size(mac,2), 1:size(mac,1));
text(1:9,1:9,diag(mac),labels,'HorizontalAlignment','center', 'VerticalAlignment','bottom','Int
title('MAC Value','Interpreter','latex')
ylabel('Estimated Mode Shapes','Interpreter','latex')
xlabel('Analytical Mode Shapes','Interpreter','latex')
axis square
set(gca,'TickLabelInterpreter','latex')
```



As expected the identified natural frequencies showed excellent consistency to the numerical counterparts. This was true for the modal damping as well, but since the damping remains unchanged for the damaged numerical models it will not be treated in depth for the purpose of this tutorial.

Structural Health Monitoring

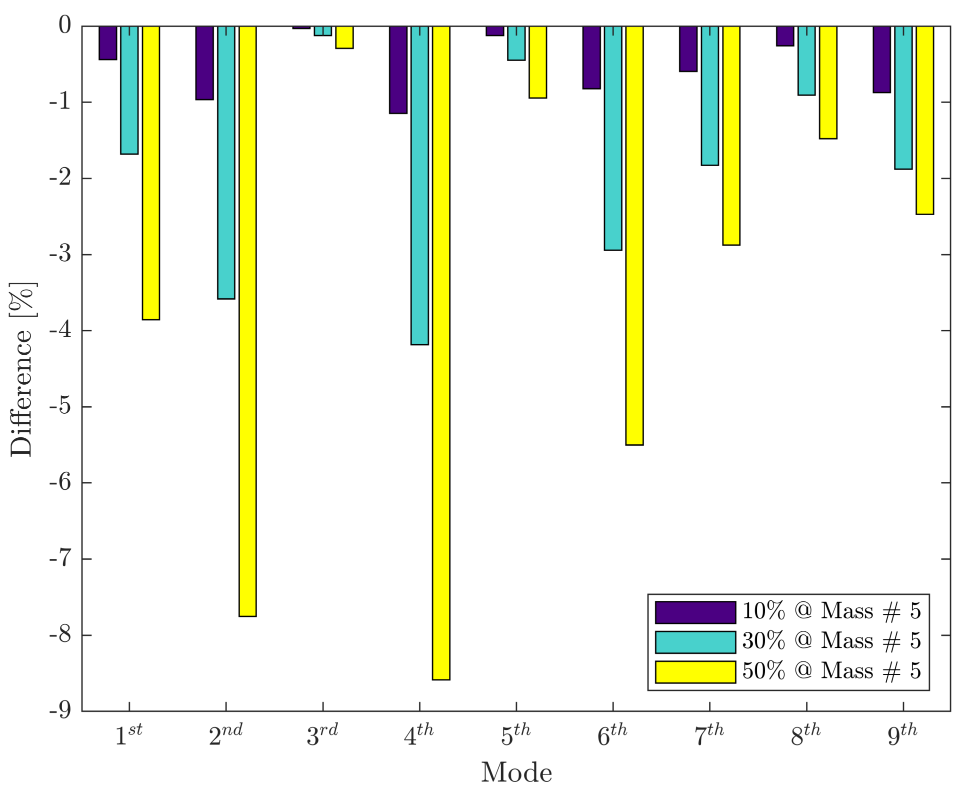
As we have assessed the capability of LF as a system identification method, we now move forward in comparing the identified modal results for the damaged model with the undamaged one.

Let's first identify the damaged dataset:

```
% 50\% damage at element 5
[id5,~,~] = LF_id(FRF(:,:,2),f,18); % This is the LF function
% 30\% damage at element 5
[id3,~,~] = LF_id(FRF(:,:,3),f,18); % This is the LF function
% 10\% damage at element 5
[id1,~,~] = LF_id(FRF(:,:,4),f,18); % This is the LF function
%let's put toghether the idenfied data
ident(:,:,1)=id.ident;ident(:,:,2)=id5.ident;ident(:,:,3)=id3.ident;ident(:,:,4)=id1.ident;
```

Let's compare the identefied natural frequencies and mode shapes in order to observe the effect of damage. For this task the raw values of thenatural frequencies are going to be plotted as the relative difference from the undamaged frequencies and the mode shapes are going to be plotted against the undamaged counterparts.

```
% natural frequencies relative difference
fr=reshape(ident(1,:,[1 4 3 2]),[size(ident,[2 3])]);
omega = 100.*(fr(:,2:end)-fr(:,1))./fr(:,1);
figure(3)
b=bar(omega);
ylabel("Difference [$\%$]",'interpreter','latex')
xlabel("Mode",'interpreter','latex')
legend({"$10\%$ @ Mass \# $5$","$30\%$ @ Mass \# $5$","$50\%$ @ Mass \# $5$"}, "location", "sou
b(1).FaceColor = [75 0 130]./255;
b(2).FaceColor = [72,209,204]./255;
b(3).FaceColor = [255,255,0]./255;
xticklabels({'$1^{st}$','$2^{nd}$','$3^{rd}$','$4^{th}$','$5^{th}$','$6^{th}$','$7^{th}$','$8^{
set(gca,'TickLabelInterpreter','latex')
```

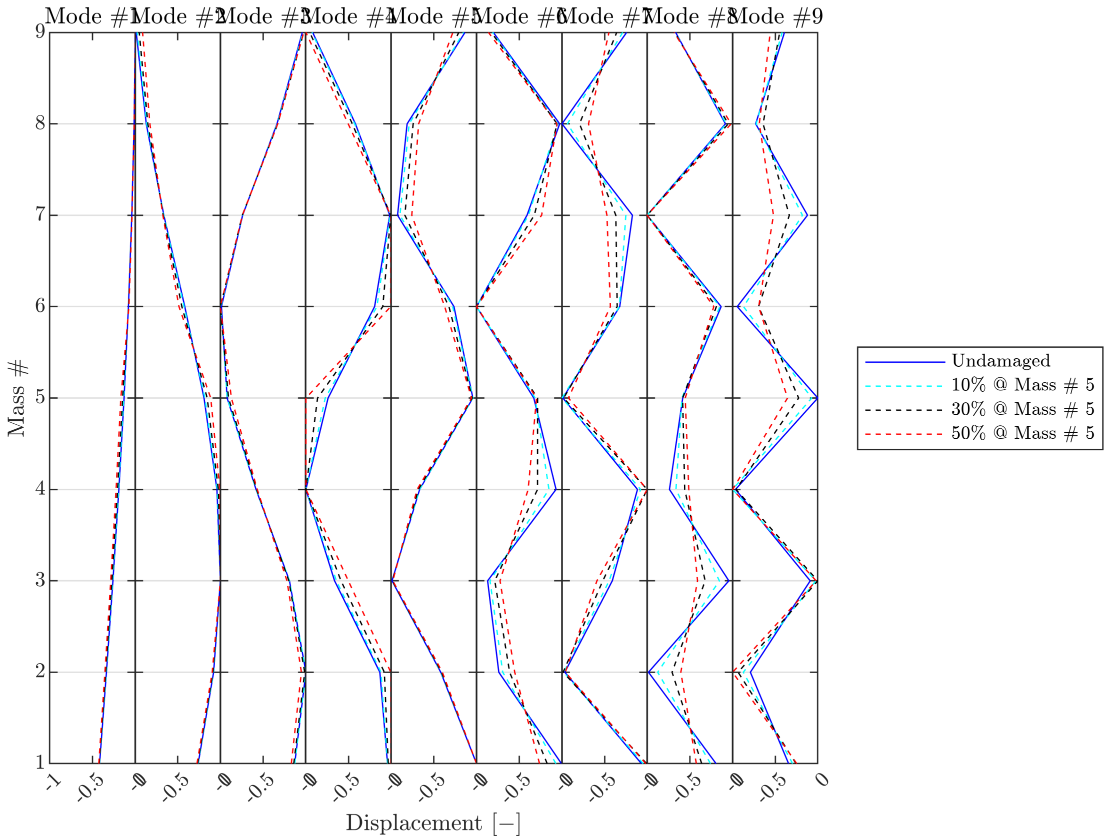


As expected, the increase in damage is accompanied by an increase in difference. This can be regarded as an indicator of damage severity.

Mode shapes, which can be used more proficiently for damage localisation, are plotted below:

```
figure(4)
t = tiledlayout(1,9,'TileSpacing','none','Padding','none');
for i=1:9
```

```
    ok=nexttile;
    p = plot(ident(3:end,i,1),1:9,'b','DisplayName','Undamaged');
    if i==5
        xlabel('Displacement $[-]$','interpreter','latex')
    end
    hold on
    xticks([-1:1:1])
    plot(ident(3:end,i,4),1:9,'c--','DisplayName','$10\%$ @ Mass \# $5$')
    plot(ident(3:end,i,3),1:9,'k--','DisplayName','$30\%$ @ Mass \# $5$')
    plot(ident(3:end,i,2),1:9,'r--','DisplayName','$50\%$ @ Mass \# $5$')
    xlim([-1 1])
    yticklabels({})
    xticklabels(['     -1' string(-.5:.5:1)])
    grid on
    set(ok, 'YGrid', 'on', 'XGrid', 'off')
    if i==1
        ylabel('Mass $\#$','interpreter','latex')
        yticks(1:9)
        yticklabels(string(1:9))
    end
    title('Mode $\#'+string(i)+'$','interpreter','latex')
    hold off
    if i==9
        lgd=legend('interpreter','latex');
        lgd.Location = 'eastoutside';
    end
    set(gca,'TickLabelInterpreter','latex')
end
```



The results in the two figures above clearly demonstrate the capability of the LFto detect even the slightest change in modal properties in a system. Hence, the LF can be used within a SHM algorithm to detect damage in structure or system.

The values presented within this tutorial might be slightly different than the ones in the original work, as the original work benefitted of higher order LF and from statistical averaging.

References

[1] G. Dessena, M. Civera, L. Zanotti Fragonara, D. I. Ignatyev, J. F. Whidborne, A Loewner-based system identification and structural health monitoring approach for mechanical systems, Structural Control and Health Monitoring, Vol. 2023 (2023). (DOI: 10.1155/2023/1891062)

[2] G. Dessena, A Loewner-based system identification and structural health monitoring approach for mechanical systems, CORD repository (2023) (10.17862/cranfield.rd.16636279)

[3] A. J. Mayo, A. C. Antoulas, A framework for the solution of the generalized realization problem, Linear Algebra and its Applications 425, pp. 634–662 (2007). (DOI:10.1016/j.laa.2007.03.008)

[4] S. Lefteriu, A. C. Antoulas, A New Approach to Modeling Multiport Systems From Frequency-Domain Data, IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems, Vol. 29, No. 1, pp. 14–27 (2010). (DOI: 10.1109/TCAD.2009.2034500)

[5] S. Lefteriu, A. C. Ionita, A. C. Antoulas, Modeling Systems Based on Noisy Frequency and Time Domain Measurements, Lecture Notes in Control and Information Sciences, vol. 398/2010, 365-368 (2010). (DOI: 10.1007/978-3-540-93918-4_33)