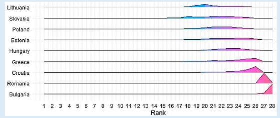
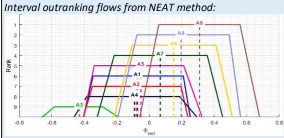
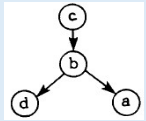
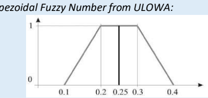
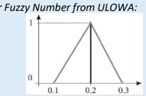
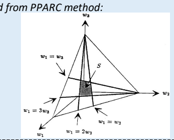
Definition of ProBCA Characterising Parameters

Section	ID	Parameter	Parameter Explanation	Comments	Available Values	Problem-Based Value Implication	Uses & Justification	Example	Comments
1. Problem Specification	**1.1**	**Task Facilitated**	*The specific task or sub-task that a DP is focused on, with reference to a generic MCDA process flow: - Options Formulation - Criteria Definition - Preference Aggregation*	*Ranking methods may or may not include sub-processes for options definition and criteria weighting. Where these sub-processes are included, they are marked as "Ranking" nevertheless, because their ultimate function is to aggregate the decision information into the global performance values.*	Ranking	The DP pursues aggregation of the decision information (option ratings and criteria weights) into a global performance value for each option to produce an overall options' ranking as core goal.	These methods are used to identify the specific position of each option in the ranking of all options. Some options may rank close to each other, but it is very rare that 2 or more options have identical global performances and occupy the same ranking position.	*Point value ranking from ARAS method:* Alt. | K | Rank A | 0.6707 | 4 B | 0.6564 | 6 C | 0.7727 | 2 D | 0.7734 | 1 ... | ... | ...	Minimum information required for a ranking operation: - Criteria-wise option performance ratings - Specific method for aggregating the data into global performance for each option (may use analytical or logical tools) Optional information for ranking: - Criteria weights (some methods do not use it)
					Criteria	Sub-activities whose sole purpose is to define the criteria and derive their importance weights.	Used to define the wording and scale of criteria that are useful for the given DP; and/or identify and represent the relative importance of criteria where it is unknown.	*Criteria weights in SWING method:* Criteria | INC | ETC | WAC | FTG | TPD | ICO Weights | 0.10 | 0.29 | 0.14 | 0.24 | 0.05 | 0.19	Subjective criteria weighting methods functionally resemble ranking that uses qualitative information. Thus, these can also be used to rank the alternatives in simple, purely subjective settings.
					Formulation	Sub-activities used exclusively to represent suitable solution options using vague knowledge about DP context and opinions distributed among its multiple stakeholders.	Used to identify suitable solution options; and/or define the description and measurement scale for its attributes. Informs the assignment of solution ratings where the ways to measure it are not readily known.	*Definition of options description in EEP method:* Group | Measures A – measures for which authority already exists. Such measures would involve minimal costs, increase awareness of key VECs, focus on impact prevention, and could be implemented quickly. | • Mark critical locations to prevent mooring near mussel beds or special shoreline areas. • Mark shallow mussel beds to reduce direct impacts of tow traffic. • Provide the navigation industry with charts showing locations of sensitive resources and include rationale for avoiding such resources.	Value naming is inspired by the used of the term in ARIADNE method source.
	1.2	**Output Format**	*The format of output values used to rank the options in accordance with DP requirements, context, constraints, or preferences.*	*Allows specifying the desired output format in line with DP requirements. E.g. for some DP, interval output is acceptable to create partial order, while precise figures and complete orders are required for other DPs.*	Point Values	The DP requires output values to take the form of numerical values.	A ranking derived using point values allows the measurement of preference intensity for each option.	*See above for "Ranking" example from ARAS method.*	Point values may take any scale, e.g. whole numbers or normalised within [0, 1] range.
					Distribution	The DP requires that output values are distributed in a non-flat pattern over a range with clear boundaries and maximum likelihood point.	Distribution is used to represent statistical outputs or probabilities.	*Distribution-type output in HSMAA method:*  	A distribution may be statistical (bell curve) or fuzzy linear (triangle) depending on the calculation method.
					Intervals	The DP requires that output values are evenly distributed over a range with clear boundaries but no indication of likely maximum.	Intervals are used to represent numerical outputs associated with uncertainty or a range of possibilities.	*Interval outranking flows from NEAT method:* 	Ranking is typically retrieved using averaging over a range in combination with context-based judgement.
					Order	The required output format represents ordinal positions without reflecting preference intensity. The order may be constructed using arbitrary point values, a sequenced list of solution references, or a mathematical graph structure acting as a visual representation of the solutions' ranking.	Allows distinguishing the more preferred alternative among any two compared, but prevents identifying the relative measure of how much more preferable one solution is to another. Mathematical graph structure offers visual presentation of results and uses relevant analytical tools to analyse the relative position of each alternative. Traditionally, the order-based ranking is produced directly by an outranking procedure. However, a large portion of methods produces an arbitrary point value to construct solution order. These values are detached from the original performance ratings of options and thus, are unable to reflect the intensity of relative option utilities in a measurable way.	*Resulting order of alternatives from ARIADNE method:*  	Where incomparability exists among the alternatives, it is technically more appropriate to use Order as an immeasurable format, rather than any measurable type. While some methods may allow producing arbitrary values, these will always have an element of imprecision and unreliability because incomparable alternatives have at least one criterion that is not common, preventing a true absolute comparison.
					Statements	The output has to be presented in the form of descriptive statements based on natural language.	Used for Criteria Definition methods that are concerned with the definition of criteria wording (which may or may not be concerned with criteria weighting), and Options Formulation methods concerned with derivation of Options Description (which may or may not be concerned with Option Performance Rating).	*See above for "Formulation" example from EEP method.*	Statements are not sufficient for a complete Ranking task and only occur in constituent stages preceding the analytical part of the MCDA task.
					Mixed	The output is presented in a mixture of formats from the list above.	Only produced by Options Formulation-type methods that are simultaneously concerned with defining options from a range of subjective opinions and evaluating its performances against criteria.	*Mixed output types in NGT method:* *Participants' ratings of five favourite ideas to promote health in our community* Good Ideas Generated | Participant Number 1 2 3 4 5 6 7 8 Ratings | Total Provide free information – free cooking class, recipes | 1 4 4 1 5 1 | 16 Healthy school lunches (healthy lunchbox competition) | 3 5 4 | 12 Advocacy materials – develop brochure/pamphlets and distribute to library, schools, churches, hospital, shops | 5 3 | 8 Health awareness Community groups/similar to neighbourhood watch ('Health Watchers') | 3 1 1 3 | 8 Competitions among various communities | 2 2 2 2 | 8	N/A
	1.3	**Ambiguity Presence**	*Indicates the presence of ambiguity i.e. when some of the quantitative decision information (criteria weights or option ratings) is unknown. This relates to cases where: - A value may exist in several different versions expressed by different DMs (multiple opinions on specific values); - Precise boundaries or some values are unknown (unknown data limits); - Some values may be absent in a range (i.e. incomplete data between limits)*	*Multiple distinct values corresponding to any option rating (dimension 1) or criteria weight (dimension 2) from multiple DMs (dimension 3) is a 3D DP, as opposed to 2D DP where only a single DM is involved. No other parameter reflects method applicability to GDM case. Some methods do not attempt to pin down ambiguous information to specific values and produce an output in interval or distribution format. Other methods, which provide a Point Value output, include instruments to pre-process imprecise ratings into crisp format before Preference Aggregation takes place. This is performed using dedicated techniques specified as a part of the method.*	Option Ratings	Ambiguity exists within the knowledge related to option performance ratings.	Various forms of ambiguity representation allow dealing with a lack of precision or knowledge in the decision information. Grey numbers are considered more objective because they only indicate the max/min boundaries, which may be easier to obtain by objective judgement of possible system states. Fuzzy numbers are presumed less objective since specifying the "likely" point tends to involve subjectivity.	*Trapezoidal Fuzzy Number from ULOWA:*  	As a general rule: - Triangular Fuzzy numbers are recognised as a distribution (min/max bounds with a vertex, which also allows a bias); - Trapezoidal Fuzzy numbers are recognised as an interval (min/max bounds, but vertex is stretched into a flat segment); - Grey numbers are recognised as interval (min/max bounds only) See the accompanying publication on ProBCA for more discussion.
					Preference Model	Ambiguity exists within the knowledge related to criteria weights, aspiration levels, and/or definition.	The same selection of formats as above (distributions, intervals) allows dealing with ambiguity in relation to criteria weights and interaction intensities (reflected by Criteria Dependency parameter).	*Triangular Fuzzy Number from ULOWA:*  	The notion of ambiguity as presented in Column E (Comments) does not match the general implication of uncertainty i.e. the absence of guarantee for the expected values to be true. The use of this parameter drives the DM to specify any "uncertain" values to reach certainty regarding their possible boundaries, multiple variants, and knowledge gaps.
					Both	Ambiguity exists within the knowledge related to both option performance ratings and criteria parameters.	A simultaneous combination of the above two types of ambiguity.	*See above for individual ambiguity examples from "ULOWA" method.*	N/A
					N/A	No ambiguity is present in the decision information.	Used for DP cases associated with complete clarity of information.	*See above for "Ranking" example from ARAS method.*	N/A

<table>
<tr><td rowspan="7"></td><td rowspan="4">1.4</td><td rowspan="4">Use of Thresholds</td><td rowspan="4">Indicates whether thresholds are used i.e. pre-defined bounds limiting the overall or relative influence of quantitative decision information (option ratings, criteria weights) within a DP context.</td><td rowspan="4">Thresholds serve to segregate influences, either between options (preference, indifference) or criteria (veto levels limiting maximum influence of any criterion). This is different form Bounds deliming the acceptable rating values to eliminate under- or over-performing options in a Selection task. Since ProBCA concentrates on Ranking, the elimination tools are left outside of its scope. All options participate in a ranking assessment, unless pre-excluded by the DM.</td><td>Option Ratings</td><td>Pre-defined value brackets controlling the relative effect of varied option performance ratings. Consider the difference in option performances against a particular criterion ("delta"):
- If delta falls within the indifference bracket, options are considered equivalent and occupy same ordinal position;
- If delta falls beyond the indifference value but within preference bracket, higher-performing option is given a higher ordinal position, which may be a measurable or an immeasurable ordinal difference depending on the method;
- If delta exceeds preference threshold, the rating of higher-performing option is replaced by the preference threshold value in any aggregation calculations to limit its total effect.</td><td>Used to define an ordinal relationship in a pairwise comparison of any two alternatives against a given criterion by delimiting the acceptable difference between their performances. Defines the "indifference" and "preference" intervals for option rating deltas. Naturally imposes a maximum "veto" limit beyond which the contribution of a particular rating in any single criterion, even an outlier, remains constant and does not affect the aggregated performance of an option too heavily.</td><td>Basic binary relations used in ORESTE and other methods:
- P = "Preference"
- I = "Indifference"
$aPb \quad \text{iff } v(a) > v(b) + \delta, \quad \delta > 0, \quad \forall a, b,$
$aIb \quad \text{iff } |v(a) - v(b)| \leqslant \delta.$</td><td>Applicable to purely ordinal tasks that use measurable ratings to derive an order of options; and to part-ordinal tasks where individual option ratings contribute to quantifying aggregated performance after pre-ordering the options using ordinal relations imposed by the thresholds.

Thresholds may be defined using some objective basis, but also offer a mechanism to impose the DM's subjective delimiters on objective data for both weights and ratings.</td></tr>
<tr><td>Criteria Influence</td><td>Pre-defined weight brackets or fixed nominal values to choose from delimiting the influence of any single criterion on the total performance aggregation.

Weight thresholds are defined as fixed values ("weights fall within [0.2; 0.8] range"), standard deviation, feasible region of a 3D graph, or in any other way.</td><td>Constrains criteria importance in cases when any single criterion weight is distinctively higher/lower than that of other criteria:
- if too high, that one criterion drives the entire aggregation while other criteria have no noticeable role in producing the final ranking, which is nearly equivalent to running a single-criterion problem;
- if too low, that one criterion effectively does not participate in forming the final ranking, which is equivalent to omitting it.</td><td>Extracted from PPARC method:

</td><td>Weight thresholds limit the impact of diverse criteria weights if they are heavily imbalanced. This may occur if criteria weighting is sourced by subjective opinions expressed through direct assignment or pairwise comparisons.

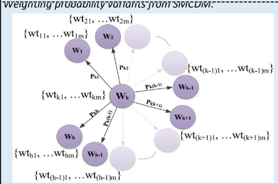
"Veto" term may indicate both criteria influence bounds and differential preference relations, causing confusion.</td></tr>
<tr><td>Both</td><td>Pre-defined brackets for both the criteria weights and the differences in pairwise option performance comparisons are applied simultaneously.</td><td>Used in cases when the DP context is prone to disbalance, which is the likely case when the decision information is heavily subjective or ambiguous (see Ambiguity parameter above).</td><td>See above for "Option Ratings" and "Criteria Influence" examples.</td><td>Rating thresholds are inapplicable when the DP only involves abstract ratings; weight thresholds are inapplicable when all criteria exhibit equivalent importance levels.</td></tr>
<tr><td>N/A</td><td>No limitations are imposed onto decision information.</td><td>Typically the case for simpler DP types that use pre-defined decision information e.g. based on objective measurements.</td><td>N/A</td><td>N/A</td></tr>
<tr><td rowspan="3">1.5</td><td rowspan="3">Resource Needed</td><td rowspan="3">The relative measure of effort, time, finance, computational power, expertise, and other resource that needs to be spent for an efficient and effective method application.</td><td rowspan="3">Not an objectively measured value; but serves as a useful tool for planning the timeframe, selecting an appropriate DM (e.g. identifying an internal capability or hiring an external facilitator), and budgeting an MCDA project.

Also reflects the quality of available literature e.g., the nature of a method may be average, but information delivery in the existing references make it difficult to understand its application intricacies.</td><td>Light</td><td>Low-complexity DP that allows independent method application by a single, non-expert DM with no advanced math skills, who uses hand calculations or a simple digital setup (e.g. MS Excel tool) utilising no advanced capabilities.</td><td>The described conditions imply the use by someone who may not have extensive analytical experience and possess no prior MCDA awareness without any additional supervision, e.g. a graduate.</td><td>N/A</td><td>Methods using mathematical functions are in most cases Light or Reasonable.</td></tr>
<tr><td>Reasonable</td><td>DP complexity level implies the use of MCDA method that requires computer-based capability and a DM with some knowledge of MCDA and maths (e.g. an engineer); and oversight by another professional of at least equivalent level for quality assurance and error proofing.</td><td>The described conditions imply that method application is excessively complex for pen-and-paper implementation, but allows a DM who possesses non-expert MCDA awareness.</td><td>N/A</td><td>N/A</td></tr>
<tr><td>Heavy</td><td>The DP requires the application of a more elaborate MCDA method involving at least one MCDA expert who is highly capable in mathematical and logical concepts, and oversight by another expert of comparable level for quality assurance and error proofing.</td><td>The described conditions may be split among a team of people e.g. an OR scientist acting as the MCDA expert, while a Data Scientist acting as an expert in mathematical logic. Automated implementation likely requires developing a dedicated software, or complex analytical models e.g. MS Excel-based tools that use advanced features. The conditions allow for a non-expert to perform the task while developing expertise under the guidance of appropriate experts and secondary proofing by another set of equivalent experts.</td><td>N/A</td><td>- Methods that involve processing logical statements tend to be assessed as Heavy due to non-trivial analysis involved.
- In some cases, "Heavy" type intrinsically indicates the quality of available source: while the method itself may require "Reasonable" effort, understanding it correctly due to the specific terminology and presentation style used in the source requires sufficient MCDA expertise. For example, a method may not offer a specific example of using its results for ranking.
- In many cases "Heavy" methods offer a higher level of output refinement (detail, precision, adjustability), which correlates with heavier resource requirement.</td></tr>
<tr><td rowspan="8">2. Criteria Definition</td><td rowspan="2">2.1</td><td rowspan="2">Criteria Count</td><td rowspan="2">The most appropriate DP size that a method should be used for, expressed as a number of criteria involved.</td><td rowspan="2">Does not impose a hard limitation on method applicability, but serves as a useful indication of whether it is more suitable for tacking smaller or larger problems (in terms of criteria count).</td><td>≤ 25</td><td>The DP involves a smaller number of criteria associated with intricate information structure and modelling details, but may need a higher level of result refinement and modelling detail. Reflects whether a DP can be modelled in 1 working day as a generic indicator of complexity. Typical examples:
- DPs that use separate, unique formulae for each criterion;
- DPs that involve measurable pairwise comparison to derive criteria weights (25 x 25 criteria = 625 comparisons, -25 diagonal elements = 600/2 due to matrix symmetry = 300 entries)</td><td>The definition uses a rough time estimate required to operate the number of options involved in a DP against all criteria. An arbitrary temporal baseline of 1 working day is chosen as the delimiter, based on the following optimistic assumptions:
- 1 working day = 8 hours = 5 hours work + 3 hours setup/admin
- 5 hours work = 300 minutes = 300 x 1-minute operations
- Retrieving & entering a data point from known set = 1 min
- Modelling an intricate operation for 1 criterion = 10+ min
Thus, "25 maximum" methods are suitable for tasks involving <300 data entries and/or <30 criteria requiring complex modelling.
The delimiter is set to 25 because pairwise comparison (popular in MCDA) of more than 25 criteria exceeds 300-minute effort baseline.</td><td>Pairwise criteria importance comparison from AHP:
<table>
<tr><th></th><th>EEE</th><th>ECE</th><th>ICE</th><th>CSE</th><th>MECH</th><th>priority vector</th></tr>
<tr><td>EEE</td><td>1</td><td>1</td><td>3</td><td>1</td><td>1</td><td>0.2284</td></tr>
<tr><td>ECE</td><td>1</td><td>1</td><td>3</td><td>1</td><td>3</td><td>0.2846</td></tr>
<tr><td>ICE</td><td>0.33</td><td>0.33</td><td>1</td><td>0.33</td><td>0.33</td><td>0.0755</td></tr>
<tr><td>CSE</td><td>1</td><td>1</td><td>3</td><td>1</td><td>1</td><td>0.2284</td></tr>
<tr><td>MECH</td><td>1</td><td>0.33</td><td>3</td><td>1</td><td>1</td><td>0.183</td></tr>
</table></td><td>Criteria & Options Count parameters are not fully separated:
- Where operations affected by criteria definition require less effort, it is marked "Unlimited" even if option-related operations are cumbersome (then, Options Count would be limited to 25).
- However, some methods explicitly state suitability for "large volume of options" despite including extensive option-related operations and simpler criteria structure. These are marked as "25 max" in criteria count to allow for unlimited options count.

Applying this type of methods to large DPs involving 0ver 25 criteria is possible, but will require the amount of time and effort that grows exponentially with increasing criteria count.</td></tr>
<tr><td>Unlimited</td><td>The DP involves a large number of criteria (over 25), where criteria-related formulation does not include complex structuring or intricate calculations. Typical examples include:
- DP that involves equal criteria importance weights;
- DP involving immeasurable pairwise comparison to derive importance-based criteria ordering.</td><td>Following the rationale for defining 25 criteria as the delimiter, "over 25 criteria" methods are not expected to consume a significant time of additional effort when a DP involves more than 25 criteria (which may reach hundreds).</td><td>See the number of criteria defined in the original reference for DIER-BCS method (example is too large in size to insert a visualisation).</td><td>Some "over-25 criteria" methods involve pairwise comparison. These are cases when criteria comparisons are objective and may be automated, so the DM does not have to assess each criteria pair individually. E.g. these may calculate some available values instead of using DM judgement.

Applying this type to small problems (25 criteria or less) is simple and requires no extra effort, but may not offer the level of refinement and depth of "25 max" methods.</td></tr>
<tr><td rowspan="2">2.2</td><td rowspan="2">Criteria Importance</td><td rowspan="2">Indicates whether a DP involves differential criteria importance values or only allows the use of equivalent criteria weighting.</td><td rowspan="2">Criteria weighting procedures attempt to make subjective weighting more objective. It always starts with the expression of DMs preferences, which then is rationalised using analytical techniques.</td><td>Weighted</td><td>The DP accepts differential values for criteria importance weighting.</td><td>In many real DP, criteria are viewed as having different weights of relative importance. This needs to be accounted for in the analytical procedure used by the method used to aid the decision.</td><td>Option performance aggregation that uses criteria weights, from CCSD method:
$d_i = \sum_{j=1}^{m} z_{ij} w_j, \quad i = 1, \ldots, n$</td><td>Where a ranking method suggests a specific approach for calculating criteria weights, it is also described as an integral part of that method in the original reference.</td></tr>
<tr><td>Equivalent</td><td>The DP does not consider dissimilar criteria importance.</td><td>Some DP treat all criteria with equal importance and do not need a dedicated analytical part for dealing with criteria weights.</td><td>A generic example from (Hwang, Yoon, 1981):
$$D = \begin{bmatrix} 2.0 & 1500 & 20000 & 5.5 & \text{average} & \text{very high} \\ 2.5 & 2100 & 18000 & 6.5 & \text{low} & \text{average} \\ 1.8 & 2000 & 21000 & 4.5 & \text{high} & \text{high} \\ 2.2 & 1800 & 20000 & 5.0 & \text{average} & \text{average} \end{bmatrix} \begin{matrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{matrix}$$
(columns X_1, X_2, X_3, X_4, X_5, X_6)</td><td>On the notion of "compensatory" methods - Criteria Importance does not reflect whether a method is compensatory or not:
- Compensatory methods imply that criteria tradeoffs are allowed i.e. the criteria are weighted;
- Non-compensatory methods imply no trade-offs, which means equivalent importance weights and an option is eliminated if its performance does not meet the minimum aspiration.</td></tr>
<tr><td rowspan="4">2.3</td><td rowspan="4">Weights Basis</td><td rowspan="4">The basis for criteria importance weighting applicable within given DP context.</td><td rowspan="4">Criteria weighting can be subjective, objective, or integrated in accordance with the source reference for MEREC method.

Independent from the choice of weights derivation tools (pairwise comparison, geometric centroid, etc.) - these are indicated by the following parameters for Criteria Weighting.</td><td>Subjective</td><td>Subjective opinion is used as the basis for criteria weights derivation in given DP context.</td><td>Used to indicate that the DM is expected to use own, subjective judgement when providing inputs for criteria weighting.</td><td>Criteria order as extracted from ROD method:
$w_1 \geqslant w_2 \geqslant \cdots \geqslant w_n > 0$</td><td>Subjective weighting does not solely imply that weights are subjectively assigned; it may apply to methods that use objective analytical tools on some subjective input e.g. order of importance.</td></tr>
<tr><td>Objective</td><td>Some external, independent data is used to derive criteria weights e.g. based on measured option performances.</td><td>Mostly used when there is a requirement to exclude as much subjectivity as possible from the DP. However, such an approach is argued to diminish the benefits offered by the MCDA methods.</td><td>Criteria weighting formula from CCSD method:
$w_j = \frac{\sigma_j \sqrt{1 - R_j}}{\sum_{k=1}^{m} \sigma_k \sqrt{1 - R_k}}, \quad j = 1, \ldots, m$</td><td>The only importance derivation approach that is truly objective is when option performances measured by technical means are used as direct input into criteria weighting. However, this analytical approach solely based on option performances does not offer any means to reflect DM's preferences.</td></tr>
<tr><td>Pre-determined</td><td>Criteria importance weights are used, but are not derived within given DP scope and thus, no particular approach to deriving weight values is indicated.</td><td>Used for DPs where criteria importance weights are readily available or the DM has their own preference for weighting method.</td><td>Extracted from ARAS method:
<table>
<tr><th></th><th colspan="6">Criteri</th></tr>
<tr><th></th><th>The amount of air per head</th><th>Relative air humidity</th><th>Air temperature</th><th>Illumination during work hours (8÷17)</th><th>Rate of air flow</th><th>Dew point</th></tr>
<tr><th></th><th>x_1</th><th>x_2</th><th>x_3</th><th>x_4</th><th>x_5^*</th><th>x_6^*</th></tr>
<tr><td>Weight of criteria – w</td><td>0.21</td><td>0.16</td><td>0.26</td><td>0.17</td><td>0.12</td><td>0.08</td></tr>
</table></td><td>Criteria importance weights are readily provided for method application; weighting source is not questioned by the method.</td></tr>
<tr><td>N/A</td><td>The DP that does not consider variable criteria importance.</td><td>Not used - refer to Criteria Importance - Equivalent.</td><td>See above for "Equivalent" example from (Hwang, Yoon, 1981).</td><td>N/A</td></tr>
</table>

2.4	**Weights Format**	*Weighting value format, reflects the format of input informing criteria importance weight definition that best suites the context of a given DP.*	*The preferred format may depend on the desired outcome, available decision information, or the intended use of criteria weight values further in method operation.*	Point	Criteria weighting input is provided as a precise, crisp value.	Used as the simplest representation of criteria weights. Typically used with Pre-Determined criteria, or requires good DP awareness in subjective weighting approaches.	*See above for "Pre-Determined" example from ARAS method.*	"Subjective - Point" combination implies the likely use of linguistic preference values whose quantification may be performed using the provided rules or altered by the DM to introduce a bias.
				Ratio	Criteria weighting input is expressed in measurable relative terms (quantifiable comparison), which may be normalised to [0, 1] range.	Normally retrieved by performing a quantified comparison of one criteria to another e.g. "A is 3x more important than B".	*See above for "Criteria Count - up to 25" from AHP.*	Ratio weights naturally impose implicit veto thresholds for criteria importance since comparison excludes disbalance e.g. "1/1000".
				Distribution	Criteria weighting input is provided as a non-flat range characterised by the vertex of maximum likelihood for the possible weight value and the less likely weight values spread out to range extremes.	Typically used in the following DP contexts: - GDM setting with a large number of individual DMs whose opinions on criteria weighting creates a distribution curve; - Ambiguous settings where the DM provides a linguistic estimation of criteria weights to process using triangular fuzzy numbers; - Iterative techniques characterised by the uncertainty in the overall balance of weights (i.e. no weights are fixed), where sifting through the possible weight values creates a distribution of results; - Weights expressed as a mean with standard deviation values where objective criteria weights are derived using option ratings.	*Fuzzy quantification of linguistic weights from DST:* Very low (VL) (0,0.1,0.3) Low (L) (0.1,0.3,0.5) Medium (M) (0.3,0.5,0.7) High (H) (0.5,0.7,0.9) Very high (VH) (0.7,0.9,1.0)	There are no methods reflecting a case where a DM would provide a subjective distribution of likely weights directly and it does not resemble a logical thinking context; however, this is not impossible and may occur in future methods. Distribution-based analysis produces values that are easier to understand since they are linked to maximum likelihood vertex. However, its realism is questionable since it is not entirely conclusive to condense a multitude of opinions and possibilities.
				Interval	Criteria weights can take any value along a flat range (i.e. no maximum likelihood) between some bounds.	Typically used in the following DP contexts: - The DM is not certain about a precise weight, but is able to delimit what the weights are not; - Used to reflect trapezoidal fuzzy numbers in ambiguous settings - Used to translate subjective comparison of relative importance of criteria in cases where there is no ambiguity about these relations	*Interval weights example from FWA method:* $u_{w_1}(w_1) = \begin{cases} w_1/0.3 & 0 \leq w_1 \leq 0.3 \\ (0.9 - w_1)/0.6 & 0.3 \leq w_1 \leq 0.9 \end{cases}$ $u_{w_2}(w_2) = \begin{cases} (w_2 - 0.4)/0.3 & 0.4 \leq w_2 \leq 0.7 \\ (1 - w_2)/0.3 & 0.7 \leq w_2 \leq 1 \end{cases}$	Interval-based analysis is not as clear to understand as a distribution because there is no indication of the more likely value within a range; however, it is more realistic since it gives equal consideration to all possible values within a range.
				Order	Criteria weighting input is provided in the form of immeasurable order of criteria reflecting their dissimilar importance. Immeasurable order reflects the position of each criterion within the order, but does not quantify the relationship between positions: one cannot say by how much rank 1 is more important than rank 2.	Used for subjective pairwise comparisons or direct ordering of criteria where the DM cannot pin down the specific values or reflect on the magnitude of importance, but can arrange the criteria from the most important to the least important in an order.	*See example above for Weights Basis parameter - "Subjective" value from ROD method.*	DP contexts where the DM can only provide importance order of criteria may benefit from criteria weighting thresholds, especially if many criteria are used. This would allow preventing the cases where criteria weights resemble a geometrical progression.
				N/A	Indicates a DP that does not consider variable criteria importance.	*Not used - refer to Criteria Importance - Equivalent.*	*See above for "Equivalent" example.*	N/A
2.5	**Definition Method**	*The approach (method or specific process) to retrieving input information for criteria wording and/or importance weights.*	*The most suitable method for providing inputs for criteria definition (wording, scales, weights, interaction) depends on DP context, such as: knowledge available to the DM, intended application of decision-aiding results, required DP model precision.* *Criteria Definition is mostly associated with eliciting quantitative criteria information: importance weights and interactions. It also describes criteria wording and scaling definition, but only uses "Assignment" value.*	Assignment	Criteria definition input is directly assigned without resorting to the use of any elicitation tools or procedures, disregarding input format (i.e. can assign criteria weight values, rank positions/order, etc.).	The simplest procedure for providing criteria definition inputs. Typically used with Pre-Determined criteria, or requires good DP awareness in subjective definition approaches.	*Direct assignment of criteria weights in DNMA method:* **Step 4.** Suppose that the experts assign the weights of criteria as: $\omega_1 = 0.29$, $\omega_2 = 0.26$, $\omega_3 = 0.16$, $\omega_4 = 0.12$, $\omega_5 = 0.09$	Assignment procedure does not imply questioning the source of information or attempting to amend or update it. It is also the only value that reflects qualitative criteria definition (wording, scaling).
				Comparison	Criteria parameters are derived using subjective comparison among criteria pairs.	Used to derive criteria weights from DM's subjective judgements. Whatever output format is used, pairwise comparison is used.	*See AHP example from Criteria Count parameter - "Up to 25" value for a measured pairwise comparison.*	Comparison produces an order of values, which may be measured (criterion rank position and influence intensity) or immeasurable (rank position that does not indicate influence intensity).
				Reference	Criteria parameters are derived using subjective comparison against some real or hypothetical reference e.g. sample criteria weights, partial order of importance, or min/max scaling range boundaries.	Used in cases when some information is available to help criteria definition other than the set of criteria itself. This could be min/max value boundaries, aspiration levels, partial weighting example etc.	*Benchmark-based SWING weighting:* Benchmark 3 0 2 1 4 0 7 0 INC 5 0 2 1 4 0 5 33.33 ETC 3 5 2 1 4 0 1 100 WAC 3 0 5 1 4 0 4 50 FTG 3 0 2 5 4 0 2 83.33 TPD 3 0 2 1 5 0 6 16.67 ICO 3 0 2 1 4 3 3 66.67	There are no possible external references for guiding the definition of criteria; reference can only exist within criteria set since all criteria participate in defining the parameters.
				Probability	Criteria parameters are derived using probability estimations, which may be subjective or objective data (e.g. measured statistics).	Sometimes the DM possesses sufficient information to estimate or objectively derive the probability of possible criteria parameters.	*Weighting probability variants from SMCDM:* 	Retrieving the probabilities of possible weights requires good awareness of the DP context or access to objective information. Probability-based weights are uncertainty-friendly due to allowing the consideration of different opinions with respective reflection of how likely they are to be true.
				Options-based	The definition of criteria weights is in some way based on option performance ratings and do not need DM's subjective judgements.	Useful in DP situations where maximum objectivity is required, which drives the exclusion of any subjective influence.	*Criteria weighting based on option ratings in IDOCRIW:* | A_1 A_2 A_3 A_4 A_5 r_{max}/r_{min} Weight R_1 0.4 0.15 0.15 0.15 0.15 2.7 0.2151 R_2 0.6 0.1 0.1 0.1 0.1 6 0.7849	Exclusion of subjectivity may diminish the benefits of MCDA methods in some DPs, whereas multi-attribute measurement is typically addressed by optimisation techniques e.g. MDO.
				N/A	Indicates a DP that does not consider variable criteria importance.	*Not used - refer to Criteria Importance - Equivalent.*	*See above for "Equivalent" example.*	N/A
2.6	**Criteria Dependence**	*Reflects the presence of a measurable interdependence between criteria.*	*Represents individual substitution rates of selected criteria i.e. adjustable dependency where the rating in one criterion may allow for measured loss in another one. May be defined for all criteria pairs or the select few.*	Independent	There is no interaction observed or defined among the criteria.	Ratings in any criterion do not affect the ratings in other criteria.	*See above for Criteria Importance parameter - "Equivalent" value example from (Hwang, Yoon, 1981).*	The majority of methods serve DPs with independent criteria.
				Interacting	Allows (although does not necessitate) dependencies between some or all criteria, which may be measured or immeasurable (affecting the thought process, but not necessarily expressed quantitatively).	While criteria weights reflect their influence on every aggregated score, Dependence reflects substitution effect among the select criteria pairs. Imposed using pre-defined interaction constants provided in addition to criteria weights.	*Intercriteria interaction values from IDRA method:* **Intercriteria information** **Criteria / Tradeoff / Credibility / Importance** **Speed/H.P.** — — = **Speed/Aesthetics** ? 1 2 **Speed/Price** 500 1 2 **Speed/Consumption** — — = **Speed/Comfort** — — = **H.P./Aesthetics** — — = **H.P./Price** 400 1 =	"Interacting" really means "interdependent". The former term is used as the available value in ProBCA since "interdependent" and "independent" look similar and thus, inflicts confusion.
2.7	**Criteria Hierarchy**	*Indicates where a DP context requires structuring the criteria in a multilevel hierarchy.*	*Hierarchical DP settings may be solved using non-hierarchical methods if criteria values are aggregated at the top level of the hierarchy; however, some methods offer dedicated tools as their integral component.*	Flat only	Criteria hierarchy is not considered within DP context.	The simplest DP types evaluate a flat list of criteria against a set of solution options.	*See above for Criteria Importance parameter - "Equivalent" value example from (Hwang, Yoon, 1981).*	If a "flat structure" method is applied to a hierarchical DP, effects imposed by hierarchy must be resolved as a precursor activity outside of method application scope using preferred approach.
				Hierarchical	The DP features a multi-level criteria structure, with relevant MCDA methods offering dedicated tools for dealing with multi-level aggregation effects.	Typical for the more complex DP types involving multiple criteria categories, e.g. technical, financial, perceptional, etc.	*See multi-level criteria structure defined in the original reference for DIER-BCS method (example is too large in size to insert a visualisation).*	Hierarchical methods allow multilevel criteria, but do not require it and they may be applied to flat-criteria DP. Tailoring it to a flat case carries error risk due to needing to omit some calculations.

<table>
<tr>
<td rowspan="7">3. Preference Aggregation</td>
<td rowspan="3">3.1</td>
<td rowspan="3">Options Count</td>
<td rowspan="3">The most appropriate DP size that a method should be used for, expressed as a number of entries to define performance ratings for all involves solution options.</td>
<td rowspan="3">Does not impose hard limitation on method applicability, but a useful indication of whether it is more suitable for tacking smaller or larger problems (in terms of option attribute count) based on how many values need to be filled into pre-determined data fields e.g. a decision matrix.</td>
<td>≤ 25</td>
<td>The DP involves a smaller number of solution options, but requires intricate operations to assess it. Reflects whether a DP can be modelled in 1 working day as a generic indicator of complexity. Typical example: a DP involving measurable pairwise comparison to rate 25 options (25 x 25 options = 625 comparisons, -25 diagonal elements = 600/2 due to matrix symmetry = 300 rating entries)</td>
<td>Defined using a similar logic to Criteria Count with reference to an arbitrary temporal basis of 1 working day allowing for 300 off 1-minute operations. An "operation" implies an activity (calculation, comparison, intricate logical procedure) and not simple entry of pre-determined data. The limit is set to 25 alternatives following a similar logic to Criteria Count parameter: pairwise comparison (popular in MCDA) of more than 25 options against given criterion exceeds 300-minute effort baseline.</td>
<td>Pairwise comparison of options from MARS method:
<table>
<tr><th></th><th>70, 7, VG</th><th>70, 14, VG</th><th>70, 21, VG</th><th>70, 7, AV</th><th>60, 7, VG</th><th>50, 7, VG</th><th>40, 7, VG</th><th>70, 7, VP</th><th>30, 7, VG</th></tr>
<tr><td>70, 7, VG</td><td>no</td><td>weak</td><td>weak</td><td>moderate</td><td>moderate</td><td>moderate</td><td>strong</td><td>strong</td><td>v. strong</td></tr>
<tr><td>70, 14, VG</td><td></td><td>no</td><td>weak</td><td>weak-mod</td><td>weak</td><td>moderate</td><td>strong</td><td>strong</td><td>v. strong</td></tr>
<tr><td>70, 21, VG</td><td></td><td></td><td>no</td><td>weak</td><td>weak</td><td>moderate</td><td>moderate</td><td>moderate</td><td>v. strong</td></tr>
<tr><td>70, 7, AV</td><td></td><td></td><td></td><td>no</td><td>very weak</td><td>moderate</td><td>moderate</td><td>moderate</td><td>v. strong</td></tr>
<tr><td>60, 7, VG</td><td></td><td></td><td></td><td></td><td>no</td><td>moderate</td><td>mod-strg</td><td>moderate</td><td>v. strong</td></tr>
<tr><td>50, 7, VG</td><td></td><td></td><td></td><td></td><td></td><td>no</td><td>weak-mod</td><td>vweak-weak</td><td>strong</td></tr>
<tr><td>40, 7, VG</td><td></td><td></td><td></td><td></td><td></td><td></td><td>no</td><td>very weak</td><td>moderate</td></tr>
<tr><td>70, 7, VP</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td>no</td><td>moderate</td></tr>
<tr><td>30, 7, VG</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td>no</td></tr>
</table></td>
<td>Options & Criteria Count parameters are not fully separated:
- Where option-related operations are cumbersome, Options Count is limited to 25 disregarding of criteria complexity (e.g. criteria definition may require less effort so set to "Unlimited")
- Some methods feature simpler criteria operations (should be Unlimited), but explicitly state suitability for a "large volume of options" while featuring extensive rating operations. These are marked "25 max" in Criteria Count, but "unlimited" options.

Applying "25 max" methods to large DPs involving 0ver 25 alternatives is possible, but will require the amount of time and effort that grows exponentially with increasing options count.</td>
</tr>
<tr>
<td>Unlimited</td>
<td>The DP involves a large number of options (over 25 and may reach hundreds), which are rated by simple data entry and require no complex operations.</td>
<td>A matrix can comfortably serve 25+ options if rating calculations do not need individual processing and may be automated e.g.:
- Option ratings are pre-determined e.g. obtained by simple measurement or database retrieval;
- A simple order of options is produced by immeasurable pairwise comparison that uses pre-determined, programmable logic.
Following the rationale for defining 25 criteria as the delimiter, methods serving "over 25" options are not expected to consume significant additional time of effort when options count grows.</td>
<td>Direct data entry adopted from ARAS method:
<table>
<tr><th rowspan="2">Options</th><th colspan="6">Criteria</th></tr>
<tr><th>x_1</th><th>x_2</th><th>x_3</th><th>x_4</th><th>x_5^*</th><th>x_6^*</th></tr>
<tr><td>A</td><td>7.6</td><td>46</td><td>18</td><td>390</td><td>0.1</td><td>5</td></tr>
<tr><td>B</td><td>5.5</td><td>32</td><td>21</td><td>360</td><td>0.05</td><td>8</td></tr>
<tr><td>C</td><td>5.3</td><td>32</td><td>21</td><td>290</td><td>0.05</td><td>11</td></tr>
<tr><td>D</td><td>5.7</td><td>37</td><td>19</td><td>270</td><td>0.05</td><td>9</td></tr>
<tr><td>...</td><td>...</td><td>...</td><td>...</td><td>...</td><td>...</td><td>...</td></tr>
</table></td>
<td>Applying this type (unlimited options) to small problems (25 options or less) is simple and requires no extra effort, but may not offer the level of refinement and depth of "25 options max" methods.</td>
</tr>
<tr>
<td>N/A</td>
<td>The DP does not involve any consideration of solution options.</td>
<td>The DP is solely concerned with criteria definition in separation from the part of MCDA task that considers solution options.</td>
<td>See example above for Criteria Count parameter - "25 Max" value from AHP method.</td>
<td>N/A</td>
</tr>
<tr>
<td rowspan="4">3.2</td>
<td rowspan="4">Attribute Nature</td>
<td rowspan="4">The intrinsic nature of involved option attributes affecting the approach to analysing the associated decision information.</td>
<td rowspan="4">The nature of attributes affects the approach to rating the performance of options against specific criteria. For example, where no quantitative measurements are available about an option it will have to be rated using some qualitative approach, which could be measured (nominal) or immeasurable (abstract).

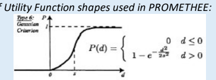
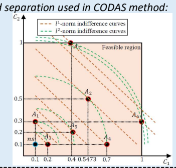
In this definition:
- Attribute = an intrinsic quality of a solution alternative that reflects its performance in the sense of a specific characteristic/aspect;
- Criterion = a measure defined by the DP within given DP context, which is used as the framework to assess a particular aspect of option performance.</td>
<td>Nominal</td>
<td>A measurable type of subjective ratings used to indicate relative option performances against any given criterion - graded order. Uses pre-defined sets of possible values with no real/material meaning, and thus are mapped on a context-dependent scale e.g.:
- Sequential scaling values (e.g. 1 to 5 scores, or 0 - 100 percentage);
- Fixed values designed to amplify the scale of preference or reflect the achievement of specific meaning/conditions (e.g. 1, 3, 7, 10);
- Linguistic values designed for use in subjective surveys, which may use pre-defined meanings (e.g. Low = 1, high = 5) or assessed within given set of results without prior definition of linguistic meanings.</td>
<td>Subjective ratings depend on the DM and may change from one respondent to another. Used to grade differential performance on attributes that are immaterial (e.g. perception, experience) or whose performance cannot be measured objectively within given DP context (e.g. comfort, suitability) by translating qualitative data into quantitative format. Implicitly reflect DM's preferences where the specific value functions are not explicitly known. Implications: for one DM or within one DP, particular features of a product (e.g. screen type, menu structure) will score higher on some criteria (e.g. comport, ease of use); whereas for another DM/DP, same features may score lower on the same criteria (comfort, ease of use) or other criteria (e.g. carrying convenience, software complexity).</td>
<td>Definition of the nominal rating scale from FMEA:
<table>
<tr><th>Rating</th><th>Description</th><th>Probability of error</th></tr>
<tr><td>10</td><td>Very frequent</td><td>Error happening at least once a month</td></tr>
<tr><td>8</td><td>Frequent</td><td>Error happening between five and ten times in a year</td></tr>
<tr><td>6</td><td>Infrequent</td><td>Error happening between once and five times in a year</td></tr>
<tr><td>4</td><td>Low</td><td>Error happening once a year</td></tr>
<tr><td>2</td><td>Almost remote</td><td>Error happening once every 5 years</td></tr>
<tr><td>1</td><td>Remote</td><td>Error happening sometimes every 5–30 years</td></tr>
</table></td>
<td>Nominal rating is not known in advance and is defined at the time of preparing the inputs for an MCDA tool. The DM assesses option attributes based on own judgement or using surveys to assign nominal scores in the most suitable format for the DP.

Nominal ratings diminish minor differences between option performances if these are smaller than scoring scale, imposing some level of imprecision when assigning a particular grade. For example, two options may perform at 45 and 55% respectively in percentage scale, but will both receive a 5 on a 1-10 score scale.</td>
</tr>
<tr>
<td>Measurable</td>
<td>Applies to DPs that involve both Nominal (relative measure) and Cardinal (absolute measure) attributes i.e. any performance that can be characterised with numerical ratings disregarding whether its basis is subjective or objective.
Cardinal measurements are a measurable type of objective option ratings that reflect option performances that are independently observed or measured using technical means. These use original measurement units and indicate the intensity of real option performance, like: dimensions (meters); weight (kg); cost (USD).</td>
<td>Measurable attributes are used in DPs that combine both subjective and objective judgements to retrieve measured ratings of option performances. A frequent case for DPs dealing with physical objects and products e.g. ranking products, equipment, transport options.
May or may not require normalisation depending on the method used. Most methods require converting quantitative data to the same scale to enable running aggregation formulae on it. See the accompanying paper on ProBCA for more on Normalisation.

On Cardinal ratings: independent observation implies that rating will not change from one observer/DM to another. These do not require rounding to the nearest score as is the case with Nominal ratings, and thus are more precise. Cardinal rating is pre-determined as an intrinsic quality of a considered option and may be retrieved using available data sources (e.g. item specification) or technical means (measurement, observation). This may be done by the DM or by anyone providing information to the DM.</td>
<td>See above for Criteria Importance parameter - "Equivalent" value example from (Hwang, Yoon, 1981).</td>
<td>For objective attributes e.g. cost, the separation of Nominal vs. Cardinal attribute type depends on DMs knowledge. For example, cost may be assessed as a nominal attribute at early project phases when the knowledge is limited and estimations are scale-based, but can be measured precisely in terms of financial values during later project phases.
Percentages may be Nominal or Cardinal depending on attribute:
- Where % assessment is defined by the DM based on their subjective judgement, it is a Nominal value;
- Where % units reflect some objective state of an alternative (e.g. the amount of material in a reservoir, it is a Cardinal measure.

Some MCDA methods are presented as if only suitable for dealing with Cardinal measurements (e.g. MADM-OPT). However, the nature of its operation does not restrict the use of Nominal scores. Generally, whenever a method can operate on measurable values, it is considered as equally suited for accepting both Cardinal and Nominal rating values.</td>
</tr>
<tr>
<td>Abstract</td>
<td>An immeasurable type of subjective ratings used to express relative position of options with respect to each other. A qualitative order indicating which option meets the DP goals better in a pair. Cannot be evaluated in terms of any numerical value of intensity. May be based on either subjective or objective qualities e.g.:
- Objective abstract: Colour (independently observed; one may be seen better than other for given DP context e.g. user group)
- Objective abstract: Direction (independently observed; one may be better than another e.g. for a construction project)
- Subjective abstract: Beliefs (subjectively perceived; one population group may be judged by the DM as having particular views about some subject e.g. a new policy that are better than the views of another group, as perceived by the DM).</td>
<td>Subjective by nature i.e. none are objectively better, whereas their order of performance depends on the specific DP context and DM goals. Abstract ratings have directions distinguishing what is better and what is worse, but offer no basis to reflect preference intensity i.e. does not allow to indicate "better by how much?". Typically use pairwise comparisons among each other or against some reference.</td>
<td>Objective units used for measurable attribute ratings in ARAS method (note: ARAS is suitable for any rating of Measurable nature, but offers an illustrative example):
<table>
<tr><th rowspan="3"></th><th colspan="6">Criteria</th></tr>
<tr><th>The amount of air per head</th><th>Relative air humidity</th><th>Air temperature</th><th>Illumination during work hours (8÷17)</th><th>Rate of air flow</th><th>Dew point</th></tr>
<tr><th>x_1</th><th>x_2</th><th>x_3</th><th>x_4</th><th>x_5^*</th><th>x_6^*</th></tr>
<tr><td>Measurement units</td><td>m^3/h</td><td>%</td><td>°C</td><td>lx</td><td>m/s</td><td>°C</td></tr>
</table></td>
<td>Whether a qualitative attribute is immeasurable (Abstract) or measurable (Nominal) depends in part on the attribute itself, and in part on the DM's attitude within given DP context. For example:
- Colour may be Abstract if different values (Red, Blue, Green) are compared by the DM in pairs as a better or a worse option for the DP, but cannot be graded by "how much better" each option is.
- Colour may be a Nominal attribute if its values along the same scale (Yellow, Orange, Red, Brown) can be measured as better or worse among each other e.g. Orange is 2x better than Yellow, Red is 3x better, Brown is 4x better. This also allows the DM to adjust the scaling e.g. Orange is 1.2x better and Red is 2.5x better.

Abstract attributes reflected with binary values (Yes/No type) can be categorised as either Abstract or Measurable depending on the DP context and DM's convenience:
- In Abstract sense, these offer a clear indication of which one is better than the other (e.g. Yes = better, No = worse);
- In Measurable sense, these may be rated using a binary set of values e.g. Yes = 1 or 100%, No = 0</td>
</tr>
<tr>
<td>Any</td>
<td>Identifies MCDA procedures whose operation is not directly linked to the format of attribute ratings and as such, can accept any type of attributes. For example, these procedures may assign a particular nominal value to the better and the worse option in a pair (e.g. 0.0 if worse, 0.5 if indifferent, 1.0 if better) following the same scale disregarding of how option performance is assessed to inform comparison (e.g. may be subjective immeasurable order, subjective nominal scoring, or an objective parameter measurement).</td>
<td>Used in versatile DPs characterised by no direct link between option qualities (observed independently or subjectively) and attribute-wise ratings (derived in relation to DP context). Such DPs typically involve various attributes representing the needs of many stakeholders. Each stakeholder typically pursues own goal, which may be characterised by the most suitable/convenient approach to assessing option performances against the attributes relevant to stakeholders' businesses.</td>
<td>Assignment of pre-defined nominal values to reflect ordinal relationship between option pairs in DRAPE:
$$t_{ij}^{W} = \sum_{k=1}^{p} w_k \cdot \delta_{ij,k} \quad \text{where} \quad \delta_{ij,k} = \begin{cases} 1 & \text{if } x_{ik} \rhd x_{jk} \\ 0.5 & \text{if } x_{ik} \triangleq x_{jk} \\ 0 & \text{if } x_{ik} \lhd x_{jk} \end{cases}$$</td>
<td>N/A</td>
</tr>
</table>

3.3	**Rating Format**	*Rating Value Format, represents the format of values expressing option performance ratings (absolute or relative) in the decision matrix.*	*Choosing the format for option rating values allows the inclusion of option performances in a particular attribute (cardinal or ordinal), the DM's risk attitude, intrinsic indication of the utility function shape, and other decision parameters.* *Option ratings express absolute data points e.g. cardinal measurements, nominal scores, probability estimations, or distributions; and relative assessment results.* *An intermediary between Attribute Nature and Rating Procedure, these depend on a combination of DM preferences and further processing approach used by the considered MCDA method.*	Point Value	Option ratings are provided as precise, crisp values reflecting absolute assessment of option performances in given attribute. Independent of other attributes and disconnected from other option ratings, even when indirect comparison is involved. Examples: - Measurement of some physical parameter (weight, size) - Assessment of performance on a pre-defined nominal scale - Assignment of a pre-determined value to reflect comparison result - Probability estimations for the various possible system states.	A simple representation of Measurable (Nominal, Cardinal) option performances. Normally requires readily available data on option attributes, or good DP awareness exhibited by the DM to produce a reliable and realistic nominal scoring.	*See above for Criteria Importance parameter - "Equivalent" value example from (Hwang, Yoon, 1981).*	In most cases, does not require any dedicated pre-processing or derivation activity other than normalisation. Therefore, more typical for methods serving "Unlimited" options count. Indirect Comparison is such when any two options are compared, a pre-determined value representing their ordinal position is entered (e.g. 0 for worse, 1 for better, 0.5 for similar) instead of recording the relationship between their absolute performances.
				Ratio	Option ratings are expressed in measurable terms to represent relative preference intensity between any two options. Suitable for measurable attribute types (Nominal or Cardinal), unless an independent quantification approach (probability or reference-based rating) is used by a particular method - in which case any attribute type works.	Represents a measurable assessment of relative performance of criteria pairs within selected criterion. Retrieved by performing a quantified comparison of one criteria to another e.g. "A is 3x more important than B". Typically normalised in [0, 1] range.	*Ratio-type ratings produced by FDMM comparison:* $P_{11} = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0 & 1 & 0 \\ 0.\overline{3} & 0.\overline{3} & 0.\overline{3} \end{bmatrix}; \quad P_{12} = \begin{bmatrix} 0.\overline{3} & 0.\overline{3} & 0.\overline{3} \\ 0 & 0.5 & 0.5 \\ 0 & 0 & 1 \end{bmatrix}$	The term "Ratio" is used to imply that values reflect the relative preference intensities between alternatives carrying no real / material meaning and thus, are dimensionless.
				Distribution	Option ratings are expressed as a non-flat range characterised by a vertex representing the most likely performance value for an option, with less likely performance values spread out to range extremes.	Typically used in the following DP contexts: - GDM setting with a large number of DMs who have individual opinions on option performances creating a distribution curve; - Ambiguous settings where the DM provides a linguistic estimation of option ratings to be processed using triangular fuzzy numbers; - Iterative techniques characterised by performance estimation uncertainty (i.e. no ratings are fixed), where sifting through the possible performance values creates a distribution of results; - Ratings expressed as a mean with standard deviations or errors.	*Distributional probability estimations from MZM:* Alternatives | $i=1$: [0, 1, 2] | $i=2$: [0, 1, 2] | $i=3$: [0, 1, 2] a | 0 0 1 | 0 0.5 0.5 | 0.3 0.4 0.4 b | 0 0.5 0.5 | 0.3 0.3 0.4 | 0.5 0.5 0 c | 1 0 0 | 0.3 0.7 0 | 0.5 0 0.5 d | 0.3 0.7 0 | 0.5 0.5 0 | 0.5 0.5 0	Distribution-type option ratings may or may not reflect the presence of triangular fuzzy numbers representing ambiguity: - Multiple Point ratings may be treated as a fuzzy distribution; - Distribution-type original rating may have no ambiguity about it; - A multitude of Distribution ratings may have a secondary set of distribution parameters representing the associated ambiguity.
				Interval	Option ratings can take any value along a flat range (i.e. no maximum likelihood vertex) defined between some specified bounds.	Used to reflect data intervals such as: - Multiple DM opinions without clear dominance of opinion (GDM); - Possible variation of option performance values within a range without a clear indication of its differential likelihood (uncertainty); - An uncontrolled multitude of possible system states (variability). Typically analysed using arithmetical (e.g. average), probabilistic, or distributive functions.	*Mixed point and interval ratings from RICH method:* | a_1 | a_2 | a_3 | a_4 | a_5 $v_i(x_1)$ | [0.80,1.00] | [0.70,0.90] | 0.80 | 0.40 | 0.70 $v_i(x_2)$ | [0.00,0.20] | [0.50,0.70] | [0.40,0.60] | [0.20,0.60] | [0.30,0.90] $v_i(x_3)$ | 0.60 | [0.50,0.70] | 0.60 | [0.20,0.40] | [0.30,0.90]	Interval option ratings may or may not reflect the presence of trapezoidal-type ambiguity: - A multitude of Point ratings may be treated as a fuzzy interval; - An Interval rating may have no ambiguity about it; - A multitude of Interval ratings may have a secondary set of boundaries representing Interval-type ambiguity.
				Order	Options are rated in the form of immeasurable order. It reflects the relative position of each option vs. each other option in terms of particular attribute, but offers no quantifiable indication of the preferential distance between the options i.e. it is impossible to say by how much Position A is better or worse than Position B.	Used for pairwise comparisons within a set or against a reference to identify the position of each option within ranking. Enables ordering the options to reflect superior performance when: - Performance ratings cannot be measured due to attribute nature, - Performance ratings are measurable, but do not directly participate in deriving an immeasurable order due to DP context.	*Unquantifiable ordinal ratings from IDCR method:* $\boldsymbol{r1}$ (easy to hold) : $a1 \sim a4 > a2 \sim a3$ $\boldsymbol{r2}$ (does not smear) : $a3 > a2 > a1 \sim a4$ $\boldsymbol{r3}$ (point lasts) : $a2 > a3 > a1 > a4$ $\boldsymbol{r4}$ (does not roll) : $a4 > a1 > a2 \sim a3$	Ordering may be subjective (DM comparing options) or objective (immeasurable order is derived based on known performance values). Ordering may be cone in pairs (comparison one by one, suitable for problems featuring 25 criteria or less) or by direct assignment of rank positions (comparison in bulk for problems featuring over 25 criteria); the former is more refined but arduous.
				N/A	Option ratings are not involved in the task addressed by the considered MCDA method.	Indicates an incomplete DP covering a part of the MCDA process, i.e. concerned with Criteria Definition or Options Formulation.	*See above in Task Facilitated parameter - "Formulation" example from EEP method.*	11 out of 15 methods serving Options Formulation include rating of option performances (but not aggregation); 4 only cover definition.
3.4	**Rating Procedure**	*The core activity involved in interpreting the attribute-wise option performance information.*	*An intermediary between Attribute Nature and Aggregation Method, it depends on a combination of available information about the attributes and aggregation approach used by the considered MCDA method. E.g. the original data may be point values, but the decision matrix contains versions of immeasurable option orders (one per criterion) as a precursor to outranking.* *It is defined as the "core" activity because there may be different parts to performance rating process (e.g. ordering, normalisation, summation, etc.) - but only one activity can be identified as the basis for the process.*	Direct Rating	Option ratings are subjectively assigned or objectively measured by the DM in a direct manner, with no other activities preceding the provision of rating values into the decision process. Uses original measurement units of performance in each attribute (if any).	Equivalent to Assignment operation for Criteria Weights. The simplest procedure for rating option performances. Typically used in DPs that feature pre-determined attribute information, or requires good DM's awareness in subjective rating approaches.	*See above for Criteria Importance parameter - "Equivalent" value example from (Hwang, Yoon, 1981).*	Direct rating does not imply questioning the source of attribute information nor attempts to amend or update it.
				Comparison	Options are rated by relative comparison to each other in pairs (thus "pairwise") in the absence of any external reference. May be subjective (DM's judgement with respect to DP goals) or objective (processing measurable attribute values).	Can be based on one parameter at a time (pairwise with respect to options) or holistic option performance (pairwise per attribute). Offers maximum granularity through individual consideration of each pair of options. Includes comparison of rating frequencies per attribute e.g. in respondent surveys. May be visualised on a chart (measurable order) or a graph structure (immeasurable order).	*See above for Options Count parameter - "25 Max" value from MARS method.*	Comparison may use attribute endpoints (Min and Max possible rating values) as reference for comparison. However, these methods are still Comparison type and not Reference because ideals are defined within the available set of options and without any additional information sourced from outside of the given set.
				Reference	Options are rated by relative comparison of options against external references (which may be one or several). May be subjective (DM's judgement with respect to DP goals) or objective (processing measurable attribute values). Reference comparison could be performed on historical ratings, partial comparison samples, hypothetical alternatives.	Used in cases when some information is available in addition to baseline set of options to serve as comparison basis. A hypothetical alternative may be defined by a set of specific goals, one per each criterion, which are not simply minimum or maximum range points but driven by some specific logic or context. Other justification points are identical to Comparison value above.	*Options rating alongside reference from ORME method:* | | A: Ref | A: Full | B: Ref | B: Full | C: Ref | C: Full 1 | kWh/m^2 | 38 | 25 | 64 | 28 | 263 | 47 2 | kWh/m^2 | −35 | −2 | 17 | 21 | 4 | 0 3 | kWh/m^2 | 45 | 31 | 87 | 87 | 80 | 59 4 | €/m^2 | — | 300 | — | 350 | — | 400 5 | kg/m^2 | 50.8 | 34.3 | 102 | 83.2 | 212 | 64.9 6 | kBq/m^2 | 568 | 383 | 1150 | 930 | 2373 | 725 7 | % | — | 5 | — | 3 | — | 20 8 | $m^3/(hp)$ | 46 | 46 | 124 | 91 | 49 | 37	Where comparison is performed against a "hypothetical" alternative that solely uses minimum and maximum points for each criterion and no additional information is provided besides the available options, it is Comparison type and not Reference.
				Probability	Option performances are rated in terms of probability estimations reflecting the possible states/outcomes of the DP. May use objective data (e.g. measured statistics) or subjective estimation.	Used when the DM has sufficient knowledge to estimate probability of possible system states, but certain outcome is unknown.	*See above in Rating Format parameter - "Distribution" value from MZM method.*	Probability-based ratings are uncertainty-friendly due to allowing the consideration of different opinions with respective reflection of how likely they are to be true.
				N/A	Option ratings are not involved in the task addressed by the considered MCDA method.	Indicates an incomplete DP covering a part of the MCDA process, i.e. concerned with Criteria Definition or Options Formulation.	*See above in Task Facilitated parameter - "Formulation" example from EEP method.*	See comment above for Rating Format parameter - "N/A" value.
3.5	**Aggregation Method**	*The type of analytical instrument used at the core of aggregation procedure for the decision information (criteria and option parameters) provided for the DP. For Criteria Weighting methods, indicates the method used or process the input data and retrieve the importance weights.*	*Offers the DM a choice of what kind of instrument to use for analysing the available decision information. May depend on the available capabilities (which could range from pen and paper to sophisticated computer software) and the available resource (see Parameter 1.5).* *Some methods utilise a mixture of tools and techniques within its aggregation module. The "core" type of analytical instrument reflects the kind of approach that defines the nature of data processing activity to allow the DM understand the key activity faced.*	Functional	The aggregation procedure is based on a mathematical relation where one input maps to one and only one output, which does not use any set reference or boundary. Typically uses simple arithmetic functions (+, -, *, /) or complex functions representing a particular decision attitude (e.g. s-shaped, convex); and does not impose any boundaries to the range of possible output values.	Functional aggregation uses a combination of mathematical tools to operate on existing information. The DM's attitude is either uninvolved, or provided beforehand in the form of fixed constants or relations. No particular references (e.g. min/max or target points) are used as the baseline for judging total performances. Combines absolute ratings so aggregation scale is unconstrained.	*One of Utility Function shapes used in PROMETHEE:*  	Functional aggregation type is mostly associated with methods based on Multi-Attribute Value or Utility Theories (MAUT/MAVT) at its core. However, may also reflect any other approach to analyse a fixed relationship between performance indicators. Also includes the application of simple statistical and probability functions.
				Separation	A subtype of Functional type aggregation methods where option ratings are treated as spatial coordinates and total performance is aggregated by functional processing of the distances between rating points and some reference. Each criterion represents an individual dimension of measurement, and the references are defined either as boundaries (min/max in each criterion) or aspirations/goals (e.g. hypothetical alternative, which may not be min/max of the range).	Separation approaches constrain the scale of aggregated ratings due to modelling relative option performances. Different from Functional type by performing negative aggregation (differences between values) rather than positive aggregation (summation, multiplication, exponent, etc.). Provides the means to expressing DM's attitude through the definition of reference points.	*Visualised separation used in CODAS method:*  	Technically, Separation-type aggregation is a variety of Functional type. It is listed as a separate parameter value due to different implications on the aggregation procedure faced by the DM, and subsequently on the resource requirements.
				Programming	Models complex relations between the options using a combination of analytical and logical functions to represent relevant decision rules and attitudes. In a programming task, one input may lead to different outputs depending on the specific DP context (e.g. setting the decision rules) and DM's goals (e.g. setting logical relations). Allows modelling complex preferences and attitudes expressed by the DM or a range of involved stakeholders. Includes: - Outranking by Elimination (simple comparison); - Outranking with Binary Relations (nuanced comparison); - Iterative procedures (running multiple converging operations); - Permutations (sifting through all possible combinations of order); - Statistical analysis: complex cases with data interactions, which fall beyond Functional type e.g. Bayesian Network for probabilities, Monte Carlo Simulation for data clouds.	Overall, used in DP settings that require highly customisable modelling approach to provide the means for expressing the DM's attitudes and relevant rules. May or may not use some form of quantifying qualitative relations, which is in most cases separated from original option performances (e.g. may assign a numerical value to reflect a superior or anterior position, but will not involve attribute units or actual values used to reflect performance). Outranking is used when differential preferences cannot be quantified. Can deal with quantifiable attributes when measurable preference intensities are not required within given DP context. Typically associated with increased resource requirement. Requires dedicated computer programming to model complex cases (a software code or an MS Excel tool that uses advanced features e.g. array functions and nested "if - then" statements); can be done as a logical process by pen and paper for simpler cases.	*A set of value function constraints from GRIP method:* $\left.\begin{array}{l} U(a) > U(b) \Longleftrightarrow a \succ b \\ U(a) = U(b) \Longleftrightarrow a \sim b \end{array}\right\} \quad \forall a, b \in A^R,$ $u_i\left(x_i^{j+1}\right) - u_i\left(x_i^j\right) \geqslant 0, \quad i = 1, \ldots, n, \quad j = 0, \ldots, \gamma_i - 1,$ $u_i(\alpha_i) = 0, \quad i = 1, \ldots, n,$ $\sum_{i=1}^{n} u_i(\beta_i) = 1.$ *Constrained modelled as weak inequalities in NAROR:* $a \succsim b \Longleftrightarrow C_\mu(a) \geqslant C_\mu(b) + \varepsilon$ with $a, b \in A'$, $(a,b) \succ^* (c,d) \Longleftrightarrow C_\mu(a) - C_\mu(b) \geqslant C_\mu(c) - C_\mu(d) + \varepsilon$ with $a,b,c,d \in A'$, $i \rhd j \Longleftrightarrow \varphi(\{i\}) \geqslant \varphi(\{j\}) + \varepsilon$ with $i,j \in G$, $(i,j) \rhd^* (l,k) \Longleftrightarrow \varphi(\{i\}) - \varphi(\{j\}) \geqslant \varphi(\{l\}) - \varphi(\{k\}) + \varepsilon$, $\varphi(\{i,j\}) \leqslant -\varepsilon$ or $\varphi(\{i,j\}) \geqslant \varepsilon$ with $i,j \in G$, $(i,j) \rhd_{Int} (l,k) \Longleftrightarrow \lvert\varphi(\{i,j\})\rvert \geqslant \lvert\varphi(\{l,k\})\rvert + \varepsilon$ with $i,j,l,k \in G$, $[(i,j),(l,k)] \rhd^*_{Int} [(r,s),(t,w)] \Longleftrightarrow \lvert\varphi(\{i,j\})\rvert - \lvert\varphi(\{l,k\})\rvert \geqslant \lvert\varphi(\{r,s\})\rvert - \lvert\varphi(\{t,w\})\rvert + \varepsilon$ with $i,j,l,k,r,s,t,w \in G$, $a(\emptyset) = 0, \quad \sum_{i \in G} a(\{i\}) + \sum_{\{i,j\} \subseteq G} a(\{i,j\}) = 1,$ $a(\{i\}) \geqslant 0, \quad \forall i \in G, \; a(\{i\}) + \sum_{j \in T} a(\{i,j\}) \geqslant 0, \quad \forall i \in G$ and $\forall T \subseteq G \setminus \{i\}$. $\quad (E^{A'}_{\varepsilon})$	Typically based on either elimination of unsuitable options (which also allows to derive their order) or by establishing some measure of relative similarity and distinction among options. May or may not involve arithmetic functions within the process, but always includes logical operations on DM judgement and DP context. Technically, Programming is a complex, multilevel subtype of the Functional type aggregation. Whether a method is Programming or Functional is assessed on an individual basis and in some cases mismatches the terminology used in the sources. E.g. a method may involve a "Linear Program" to solve, but uses a set of 1D functions so categorised as Functional by the adopted convention.
				N/A	Option ratings and criteria weights are not involved in the task addressed by the considered MCDA method.	Indicates an incomplete DP covering qualitative MCDA process part, i.e. concerned with Criteria Definition or Options Formulation.	*See above in Task Facilitated parameter - "Formulation" example from EEP method.*	See comment above for Rating Format parameter - "N/A" value.