A Kriging Approach to Model Updating for Damage Detection

G. Dessena, D.I. Ignatyev, J.F. Whidborne and L. Zanotti Fragonara

Cranfield University

4th July 2022, 10th EWSHM - Palermo (Italy)

www.cranfield.ac.uk

Outline

1. Motivation
2. Background
3. Methods
4. Numerical System
5. Results
6. Conclusions
7. Bibliography

A Kriging Approach to Model Updating for Damage Detection

Main Contributions

1. Development of a refined surrogate-based single objective optimisation routine.
2. Application of the newly developed technique to numerical systems for finite element model updating and damage detection

Motivation

- Finite Element Models (FEMs) hardly ever represent correctly a given real system. Hence, some sort of tuning is always needed
- Traditionally, sensitivity, Monte-Carlo, and other iterative methods are used
- Iterative methods can be inefficient when the search direction is random, or pseudo-random
- Response Surface Models (RSMs) offer an opportunity to strategically tune the model
- Some existing RSM can have limited search capabilities

Background

Finite Element Model Updating

FEM updating is the calibration of FEMs using experimental data

Direct

- Matrix updates
- Optimal matrix
- Eigenstructure assignment

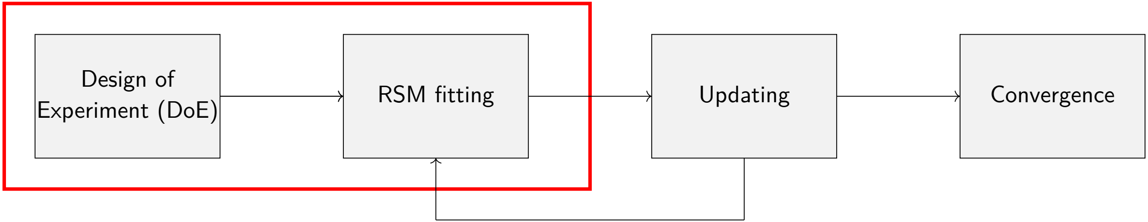
Indirect

- Sensitivity-based
- Response surface methods (RSM)
- Bayesian-Monte Carlo
- Computational intelligence
- Evolutionary algorithms

Background

Response Surface Model

- General idea: creating a response surface which mimics the relation between a function, or problem, and its input variables
- The RSM model can be a simple function, such as a polynomial, or more refined, such as Kriging



Background

- Efficient Global Optimization (EGO) is an RSM based on Kriging
- Kriging is a surrogate model based on a stochastic process

$$\hat{y}(\boldsymbol{x}_i) = \boldsymbol{f}^T(\boldsymbol{x}_i)\hat{\beta} + z(\boldsymbol{x}_i), \quad i = 1, 2, \ldots, n \tag{1}$$

Background

Based on the Expected Improvement (EI):

$$
\begin{aligned}
E[I(x)] =&(y_{min} - \hat{y}(\boldsymbol{x}))\left[\frac{1}{2} + \right.\\
&\left. + \frac{1}{2}\mathrm{erf}\left(\frac{y_{min} - \hat{y}(\boldsymbol{x})}{\hat{s}\sqrt{2}}\right)\right] + \\
&+ \hat{s}\frac{1}{\sqrt{2\pi}} exp\left[\frac{-(y_{min} - \hat{y}(\boldsymbol{x}))^2}{2\hat{s}^2}\right]
\end{aligned}
\tag{2}
$$

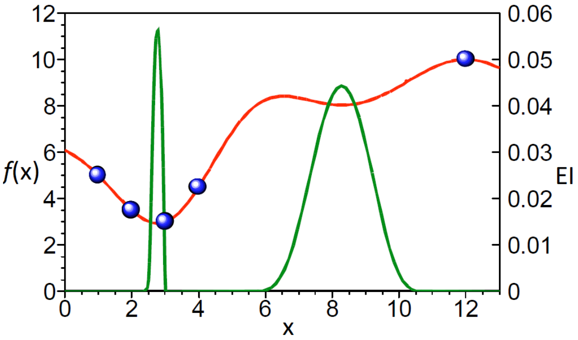
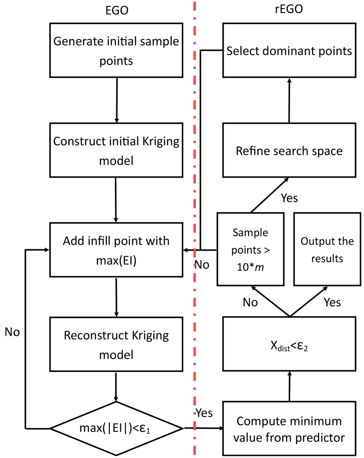


Figure: EI function, one dimensional function ($f(x)$) and known values of $f(x)$ (Adapted from [1])

Methods

- Main drawback of EGO is its global-only search capability
- refined Efficient Global Optimisation (rEGO) introduces selection and refinement techniques
- Two stopping criterion: a global one, EI, that triggers search space reduction and data points selection, and a local one based on input paramaters' Euclidean distance
- These make rEGO a global-local method, rather than only a global one



Methods

The first stopping criterion is set to 0.1% and the second to 10^{-4}

Search space refinement

- Triggered by first stopping criterion, EI based
- Ensured by a minimum number of points constraint (m x 10)
- If original search bounds are between [0,0] and [1,1] and relative minimum at [0.25,0.75] then the new interval: [0,0.5] and [0.5,1]

Selection

- The values which variables are outside of the new search space are excluded
- Further, if total number of values is more than m x 10
- Points are de-clustered, such that they are well-spaced in the search domain

Methods

FEM Updating Routine

rEGO is used to optimise a modal metric wrt stiffness and mass parameters

- Let us consider a simple 2 DoF system:

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{Bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} = 0 \tag{3}$$

where

$$m_n = x_n \times m_n^{(\mathrm{b})} \;\; \& \;\; k_n = x_{n+4} \times k_n^{(\mathrm{b})} \tag{4}$$

Where $^{(\mathrm{b})}$ stands for baseline

Methods

The goal is to minimise the residuals of the modified total modal assurance criterion ($\mathrm{MTMAC}_{\mathrm{res}}$) [2] to a set of modal parameters from a damaged system

$$\mathrm{MTMAC}_{\mathrm{res}} = 1 - \prod_{i=1}^{n} \frac{\mathrm{MAC}(\phi_i^E, \phi_i^N)}{\left(1 + \frac{|\omega_i^N - \omega_i^E|}{|\omega_i^N + \omega_i^E|}\right)} \tag{5}$$

as $\mathrm{MTMAC}_{\mathrm{res}}$ approaches zero the correlation increases

Numerical System

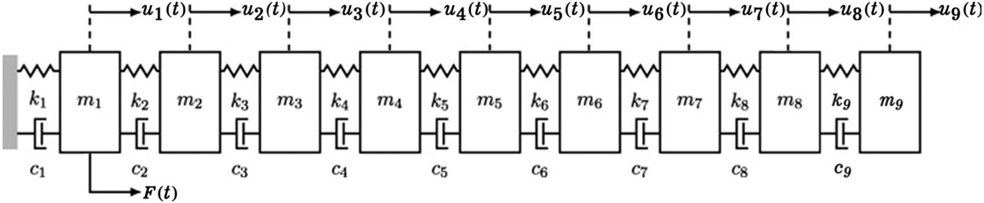


Figure: 9 DoF system. (Retrieved from [3])

Usual formulation for mass-spring-damper system, where $m_{1-9} = 1$ kg, $k_{1-9} = 10$ kNm^{-1} and $\zeta_{1-9} = 1\%$

Numerical System

Scenario #	
1	Undamaged
2	10% stiffness reduction in the fourth element
3	25% stiffness reduction in the fourth element
4	25% stiffness reduction in the fourth element and 10% stiffness reduction in the seventh element
5	25% stiffness reduction in the second element, 10% stiffness reduction in the fourth element and 10% stiffness reduction in the seventh element

Numerical System

- Scenario # 1 is taken as the reference for the updating of the model
- The assumption is that the scale of x_n corresponds to the measure of the damage in % at the n element
- Only k_n are updated in this work
- Results from rEGO are compared to:
  - Theoretical (Actual)
  - EGO
  - Genetic algorithm (GA) (MATLAB's standard with $f_{\text{tol}} = 10^{-4}$ and Max generation $= 100$)
- 10 evaluations for each case and method are taken into consideration
- Same DoE results, from a Morris–Mitchell Optimal Latin Hypercube, are used for each method

Results

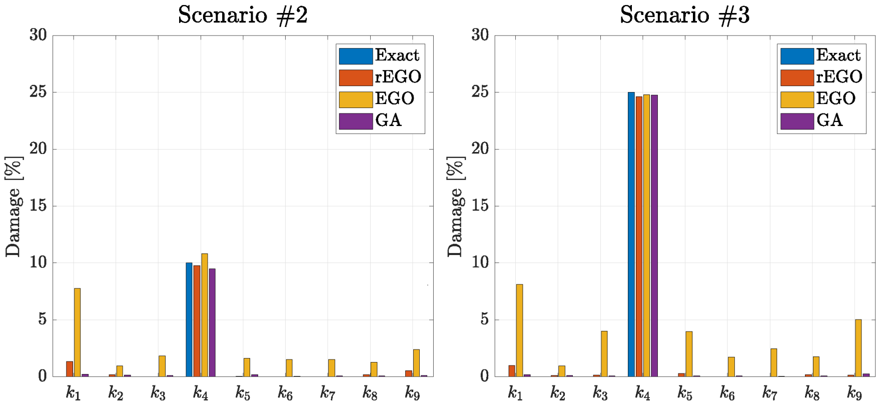


Figure: The mean values, over 10 realisations, of the identified damage in Scenarios # 2 and 3 by rEGO, EGO, and GA vs the exact value .

Results

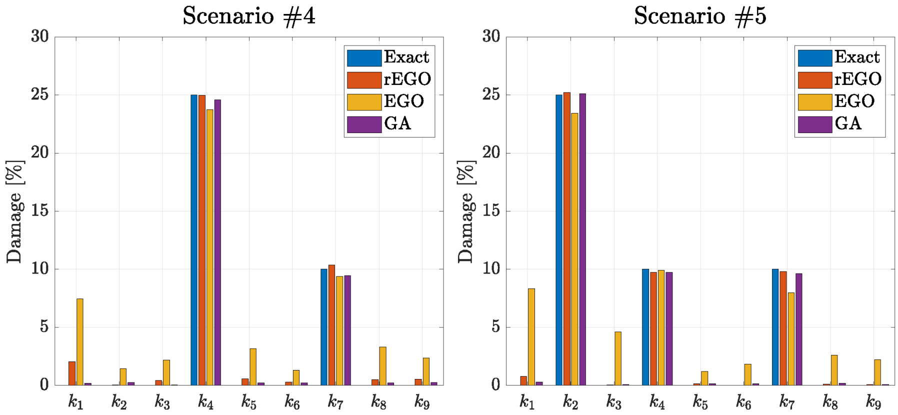


Figure: The mean values, over 10 realisations, of the identified damage in Scenarios # 4 and 5 by rEGO, EGO, and GA vs the exact value.

Results

Table: Number of function evaluations for convergence.

Scenario	#2			#3			#4			#5		
	min	μ	**max**	**min**	μ	**max**	**min**	μ	**max**	**min**	μ	**max**
rEGO	159	281	382	172	286	494	274	329	391	217	331	433
EGO	104	115	131	100	111	130	103	112	129	100	106	119
GA	15600	17899	19210	15410	17424	19210	15030	16778	19210	13700	16778	19210

- rEGO performs better than EGO
- GA performance is comparable to rEGO
- rEGO needs two order of magnitude less evaluations than GA and about 3 times EGO

Conclusions

- A new single objective optimisation technique based on EGO was introduced
- rEGO, using the $MTMAC_{res}$, successfully detected damage in a numerical system
- rEGO offered a good balance between precision, wrt EGO, and performance, wrt GA
- rEGO can be used in other engineering applications, such as Computational Design

Acknowledgements

The authors would like to thank the Engineering and Physical Sciences Research Council (EPSRC) for funding this research through grant number 2277626.

Bibliography

[1] D. R. Jones, M. Schonlau, and W. J. Welch, "Efficient global optimization of expensive black-box functions," *Journal of Global Optimization*, vol. 13, pp. 455–492, 1998. DOI: 10.1023/A:1008306431147.

[2] R. Perera and R. Torres, "Structural damage detection via modal data with genetic algorithms," *Journal of Structural Engineering*, vol. 132, no. 9, pp. 1491–1501, 2006, ISSN: 0733-9445. DOI: 10.1061/(ASCE)0733-9445(2006)132:9(1491).

[3] G. Dessena, M. Civera, L. Zanotti Fragonara, D. I. Ignatyev, and J. F. Whidborne, "A Loewner-based system identification and structural health monitoring approach for mechanical systems," *Under Review [Structural Health Monitoring]*, p. 17, 2022.

Questions

If you have any suggestions or further questions then please contact me via email at `Gabriele.Dessena@cranfield.ac.uk`.

Email

Presentation Download

Cranfield University

www.cranfield.ac.uk

T: +44 (0)1234 750111

@cranfieldduni

@cranfieldduni

/cranfieldduni